Homework 5
Due: April 11, 2011

1. *Inner bounds of DM-IC*

   For a discrete memoryless interference channel (DM-IC), the Han-Kobayashi (HK) achievable rate region is the convex closure of the rate tuple satisfying

$$
\begin{aligned}
R_{11} &\leq I(W_1; Y_1|U_1, U_2, Q)\\
R_{10} &\leq I(U_1; Y_1|W_1, U_2, Q)\\
R_{20} &\leq I(U_2; Y_1|U_1, W_1, Q)\\
R_{11} + R_{10} &\leq I(U_1, W_1; Y_1|U_2, Q)\\
R_{11} + R_{20} &\leq I(U_2, W_1; Y_1|U_1, Q)\\
R_{10} + R_{20} &\leq I(U_1, U_2; Y_1|W_1, Q)\\
R_{11} + R_{10} + R_{20} &\leq I(U_1, W_1, U_2; Y_1|Q)
\end{aligned}
$$

   and 7 similar inequalities for R_{22}, R_{20}, R_{10} for some $(p(q)p(u_1|q)p(w_1|q)p(u_2|q)p(w_2|q)p(x_1|u_1, w_1, q)$ $p(x_2|u_2, w_2, q))$. (U_1, W_1, U_2, W_2) are auxiliary random variables serve to carry the messages $(M_{10}, M_{11}$, $M_{20}, M_{22})$, respectively.

   On the other hand, the Chong-Motani-Garg (CMG) achievable rate region is the convex closure of the rate tuple satisfying

$$
\begin{aligned}
R_{11} &\leq I(X_1; Y_1|U_1, U_2, Q)\\
R_{11} + R_{10} &\leq I(X_1; Y_1|U_2, Q)\\
R_{11} + R_{20} &\leq I(U_2, X_1; Y_1|U_1, Q)\\
R_{11} + R_{10} + R_{20} &\leq I(X_1, U_2; Y_1|Q)
\end{aligned}
$$

   and 4 similar inequalities for R_{22}, R_{20}, R_{10} for some $(p(q)p(u_1, x_1|q)\ p(u_2, x_2|q))$.

   Show the equivalence between these two representations of the achievable rate regions.

2. *Gelfand-Pinsker theorem.*

   Gelfand-Pinsker theorem gives the capacity for the discrete memoryless channel (DMC) with discrete memoryless state available noncausally at the encoder as

$$
C = \max_{p(u|s), x(u,s)} (I(U; Y) - I(U, S))
$$

   Prove the converse of this theorem and explain why we can use a deterministic mapping $x(u, s)$.

3. *MMSE estimation via writing on dirty paper*

   Consider the additive noise channel with output (observation)

$$
Y = X + S + Z
$$

   where X is the transmitted signal and has mean μ and variance P, S is the state and has zero mean and variance Q, and Z is the noise and has zero mean and variance N. Assume that X, S, and Z are uncorrelated. The sender knows S and wishes to transmit a signal U, but instead he transmits X such that $U = X + \alpha S$ for some constant α

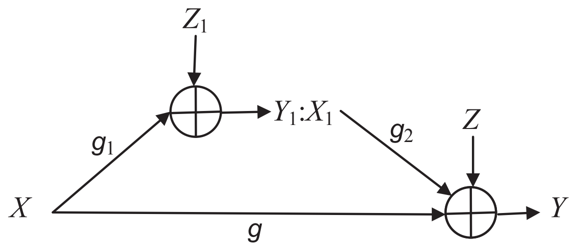
(a) Find the mean squared error (MSE) of the linear MMSE estimate of U given Y in terms only of μ, α, Q, P and N.

(b) Find the value of α that minimizes the MSE in part (a).

(c) How does the minimum MSE you obtained in (b) compare to the MSE of the best linear MSE estimate when there is no state at all, i.e., $S = 0$? Interpret the result.

4. *AWGN Relay Channel*

Figure 1 shows the AWGN-RC where $g, g_1, g_2 > 0$ are the channel gains, Z, Z_1 are iid $\sim N(0, 1)$, and X and X_1 have the same power constraint P.



(a) From the rate expressions for the discrete memoryless RC, derive the achievable rate of this Gaussian channel for decode-forward and compress-forward coding schemes. For decode-forward, show that joint Gaussian input is optimal.

(b) Using Matlab, plot the rates derived in part (a) versus P for the following cases:

i. $g = g_1 = 1$, $g_2 = 3$

ii. $g = g_2 = 1$, $g_1 = 3$

iii. $g_1 = g_2 = 3$, $g = 1$

iv. $g_1 = g_2 = 1$, $g = 3$

5. *Compress-Forward Lower Bound*

For the relay channel, the compress-forward lower bound is given as

$$C \geq \max_{p(x)p(x_1)p(\hat{y}_1|y_1,x_1)} \min\left(I(X, X_1; Y) - I(Y_1; \hat{Y}_1|X, X_1, Y), I(X; Y, \hat{Y}_1|X_1)\right)$$

Starting from the error events given in El Gamal-Kim lecture notes page $(17 - 43)$, derive this lower bound using joint typicality lemma. Show all steps in your derivation.