Homework 4
Due: March 14, 2011

1. **Two-way channel.** The 2-way channel is a channel very similar to the interference channel, with the additional provision that sender 1 is attached to receiver 2 and sender 2 is attached to receiver 1, as shown in Figure 1. Hence, sender 1 can use information from previous received symbols of receiver 2 to decide what to send next.

   Consider the 2-way channel shown in Figure 1. Assume here that the outputs Y_1 and Y_2 depend *only* on the current inputs X_1 and X_2.

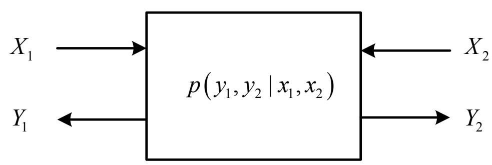


Figure 1: Two-way channel.

   (a) By using independently generated codes for the two senders, show that the following rate region is achievable:

$$R_1 < I(X_1; Y_2|X_2),$$
$$R_2 < I(X_2; Y_1|X_1),$$

   for some product distribution $p(x_1)p(x_2)p(y_1, y_2|x_1, x_2)$.

   (b) Show that the rates for any code for a two-way channel with arbitrarily small probability of error must satisfy

$$R_1 \le I(X_1; Y_2|X_2),$$
$$R_2 \le I(X_2; Y_1|X_1),$$

   for some joint distribution $p(x_1, x_2)p(y_1, y_2|x_1, x_2)$.

   The inner and outer bounds on the capacity of the two-way channel are due to Shannon. He also showed that the inner bound and the outer bound do not coincide in the case of the binary multiplying channel $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y}_1 = \mathcal{Y}_2 = \{0, 1\}, Y_1 = Y_2 = X_1X_2$. The capacity of the two-way channel is still an open problem.

2. **Superpositioned constellations** Show a superposition bit-mapping of a 16-QAM constellation on top of a QPSK constellation. Does your mapping have any special property?

3. **The Blackwell broadcast channel.** The Blackwell broadcast channel is defined as follows.

$$\mathcal{X} = \{0, 1, 2\}, \mathcal{Y}_1 = \mathcal{Y}_2 = \{0, 1\},$$
$$p(0, 0|0) = p(0, 1|1) = p(1, 1|2) = 1.$$

   (a) Is this channel degraded? Why or why not?

(b) Propose an achievable rate region for private messages using:

i. Superposition coding

ii. Marton's inner bound (over binning)

(c) Find the Sato outer bound for this channel.

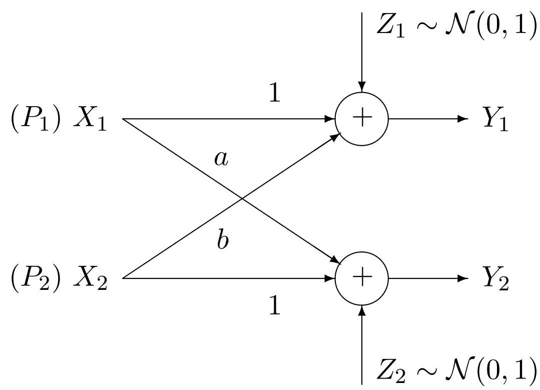
(d) Plot all three bounds on a graph. Do you have any observation on these bounds and the channel capacity?

4. **AWGN-IC.** Consider the following AWGN interference channel (AWGN-IC) model. At time i

$$Y'_{1i} = g_{11}X'_{1i} + g_{21}X'_{2i} + Z'_{1i}$$
$$Y'_{2i} = g_{12}X'_{1i} + g_{22}X'_{2i} + Z'_{2i},$$

where Z'_{1i} and Z'_{2i} are discrete-time white Gaussian noise processes with average power N_1 and N_2 respectively, independent of X'_{1i} and X'_{2i}, and $g_{jk}, j, k = 1, 2$, are the channel gains. Let the codewords X_1^n and X_2^n have average power constraints of P'_1 and P'_2, respectively.

(a) Convert the above channel to the standard AWGN-IC shown below with direct link gains of 1 and unit noise variances. What are the new codeword power constraints P_1 and P_2, and the new channel gains a and b? Provide arguments to why these two channels have the same capacity.



(b) For $a < 1$ and $b < 1$, derive the achievable rate regions for the standard AWGN-IC with the following coding schemes:

i. Time-sharing with power control

ii. Treating interference as Gaussian noise

iii. Simultaneous decoding of both messages at both receiver

iv. Han-Kobayashi rate splitting

Compare these rate regions. Does any of these regions include other regions? Prove or provide numerical examples (show a plot).

5. **Binary interference channel.** Consider the following binary interference channel:

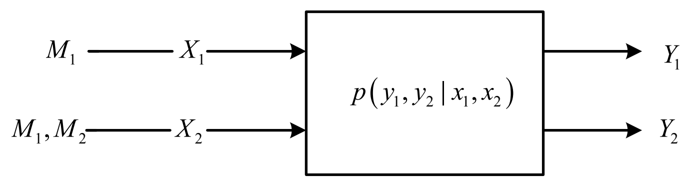
$$Y_1 = (X_1 \oplus Z_1) \cdot X_2$$
$$Y_2 = (X_2 \oplus Z_2) \cdot X_1$$

where the inputs X_1 and X_2 are binary and independent of each other, the noises Z_1 and Z_2 are i.i.d. with distribution Bern(α) and are independent of the inputs.

(a) Find the maximum rate of user 1.

(b) Evaluate the Han-Kobayashi region for this channel, using binary auxiliary random variables. (Note that binary auxiliary random variables may not be optimal, but are assumed here for simple computation.) Specify the encoding and decoding techniques and the optimal distributions of the auxiliary random variables that are used to achieve the boundary of this region.

6. **Interference channel with degraded message sets (IC-DMS).** The interference channel with degraded message set (IC-DMS) is an interference channel in which the message and codeword of one encoder is known non-causally at the other encoder. In other words, transmitter 2 encodes two independent messages (M_1, M_2) uniformly distributed over $[2^{nR_1}] \times [2^{nR_2}]$, while transmitter 1 encodes the message M_1 only. Receiver 1 needs to decode only M_1 and receiver 2 needs to decode only M_2. This channel is also called the *cognitive channel*.



Interference channel with DMS.

(a) Use Gel'fand-Pinsker and Han-Kobayashi coding techniques, propose an achievable rate region for this channel.

(b) What is this achievable rate region for the standard Gaussian IC with degraded message sets?